Monte Carlo Analysis of Uncertain Digital Circuits

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Abstract of the Full Version of the Paper

Unlike the classical deterministic digital circuit analysis, we consider a Monte Carlo simulation in the context of uncertain digital circuits. In other words, given a binary function of n uncertain input binary variables, we express the probability of this binary function in terms of the probabilities of the corresponding input binary variables. This in turn, allows us to estimate appropriate probabilistic measure of the output of a digital circuit with uncertain input parameters.

The theory of deterministic digital circuits has been studied extensively — See [4] for example. Typically, a binary variable x_j is allowed to take only two values; 0 and 1. A binary function of n binary variables $f(x_n, x_{n-1}, \ldots, x_1)$, is also allowed to take only the values 0 and 1.

The introduction of uncertainty in digital circuits has been used in different areas to model complex systems. For example, see [6] for the introduction of probabilistic Boolean networks to model gene regulatory networks. In such a model, the x_j's represent the state of gene j, where $x_j = 1$ denotes the fact that gene j is expressed and $x_j = 0$ means it is not expressed. The binary function $f_j(x_n, x_{n-1}, \ldots, x_1)$, on the other hand, is referred to as a predictor, and is used to determine the value of x_j in terms of some other gene states.

In this paper, we consider the case when the binary variable x_j is a random variable. Thus, if we consider a binary function of the n random binary variables $f(x_n, x_{n-1}, \ldots, x_1)$, then this in turn would be a random binary variable. In this context, all the variables under consideration are Bernoulli random variables since they take only the values 0 and 1.

Hence, throughout this paper, we consider the case when the x_j's are independent random variables with probabilities $P(x_j = 1) = p_j = E[x_j]$. Next, we consider the probability or expectation

$$\begin{aligned} \mathcal{P} &\doteq P(f(x_n, x_{n-1}, \ldots, x_1) = 1) \\ &= E[f(x_n, x_{n-1}, \ldots, x_1)]. \end{aligned}$$

We then, pose the following questions: Given a logic function, $f(x_n, x_{n-1}, \ldots, x_1)$, with known probabilities x_j's, what can we say about the probability $\mathcal{P}$? How can we address the problem of maximizing

or minimizing $\mathcal{P}$? The latter may refer to best case and worst case scenarios.

The flow of the full paper is as follows. in Section 2, we present and prove a result that expresses the probability $\mathcal{P}$ in terms of the probabilities p_j's, with $j = n, n-1, \ldots, 1$. The result is given in the following theorem.

Theorem 1: *Let $f(x_n, x_{n-1}, \ldots, x_1)$ be a binary function of n independent binary random variables with $P(x_j = 1) = p_j$. Let I be the set of minterm indices for which $f(x_n, x_{n-1}, \ldots, x_1)$ is 1. Then*

$$\mathcal{P} = \sum_{i \in I} \prod_{j=1}^{n} P(x_j = \lfloor i2^{-j+1} \rfloor - 2\lfloor i2^{-j} \rfloor). \tag{1}$$

Section 3 answers the question of maximizing and minimizing $\mathcal{P}$. It defines what an *essential binary variable* is, and presents and proves the following theorem.

Theorem 2: *Let $\mathcal{P}$ be a function of some p_j's. Then, for the case of maximizing $\mathcal{P}$, if the variable x_k is essential, then pick $p_k = p_k^-$ when $s_k = -1$, and pick $p_k = p_k^+$ when $s_k = 1$.*

For the case of minimizing $\mathcal{P}$, if the variable x_k is essential, then pick $p_k = p_k^+$ when $s_k = -1$, and pick $p_k = p_k^-$ when $s_k = 1$.

If x_k is not essential, then for either case pick $p_k = p_k^-$ or $p_k = p_k^+$.

We present a numerical example in Section 4 to illustrate the ideas of this paper. Finally, a summary and a suggestion of further research directions is presented in Section 5. References that are used in the paper are as follows.

References

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